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DSST ORBIT DETERMINATION IN OREKIT

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Orekit Day

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AGENDA

- Context and target
- DSST presentation
- Mean Elements derivatives
- State Transition Matrices
- Short-periodic terms derivatives
- Orbit Determination
- Conclusion

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CONTEXT AND TARGET

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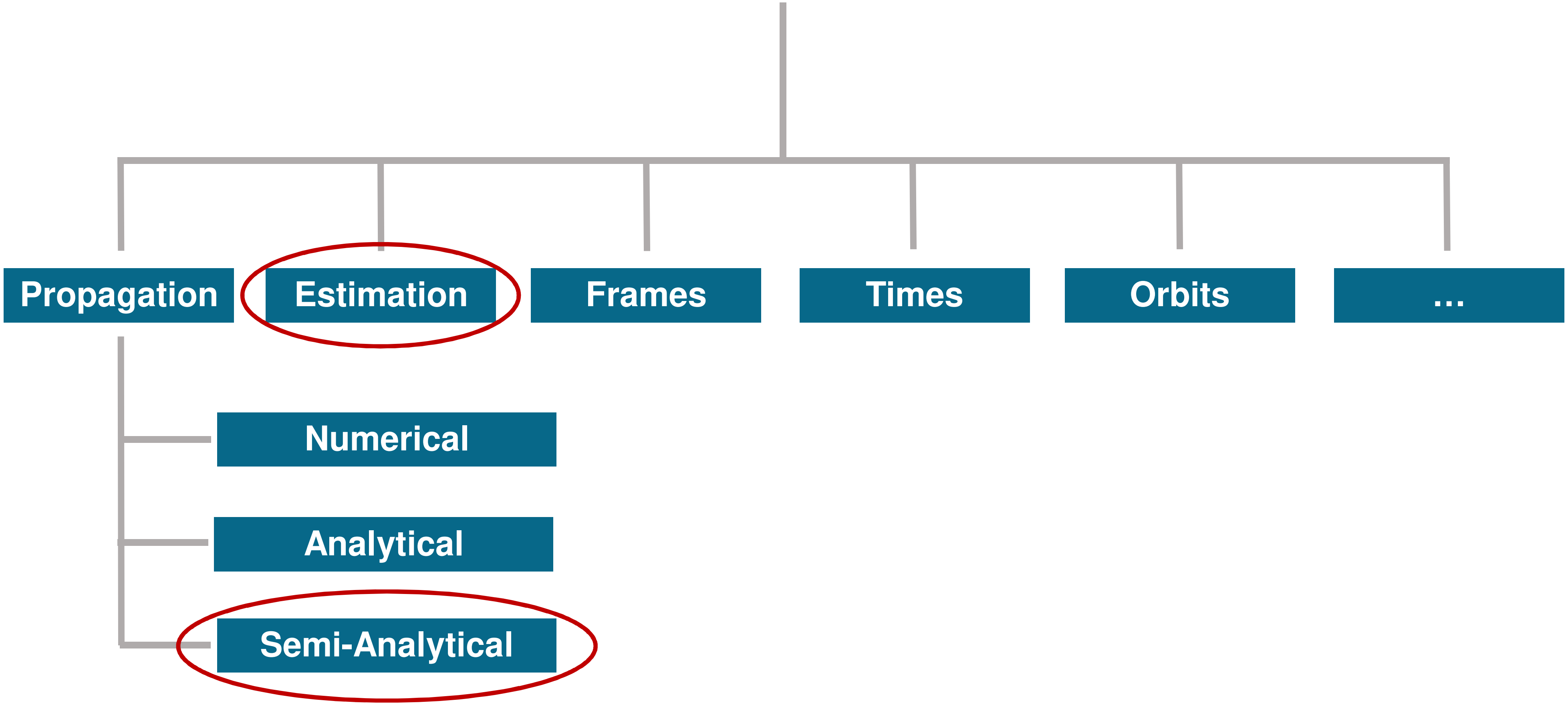
OREKIT FEATURES



TWO USE CASES

➔ Fast Orbit Determination

- **Several hundreds of thousands (Setty et al, 2016)**

Number of Orbit Determinations performed by the US Joint Space Operation Center per day to maintain their space objects catalog.

➡ **Need fast and accurate Orbit Determination**

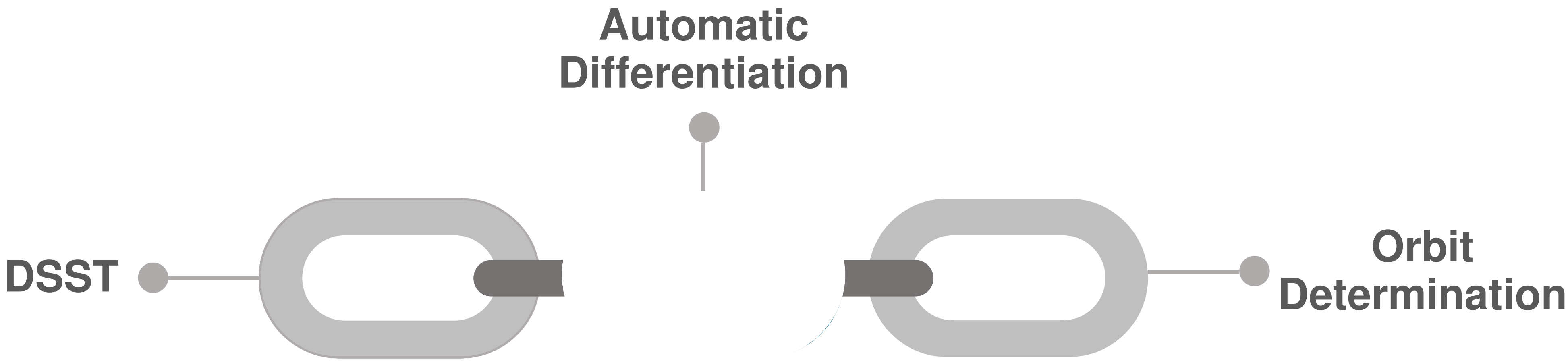
➔ Mean Elements Orbit Determination

➡ **Station keeping needs**

➔ Draper Semi-analytical Satellite Theory (DSST)

- **Rapidity** of an analytical propagator
- **Accuracy** of a numerical propagator

➔ Target



DSST PRESENTATION

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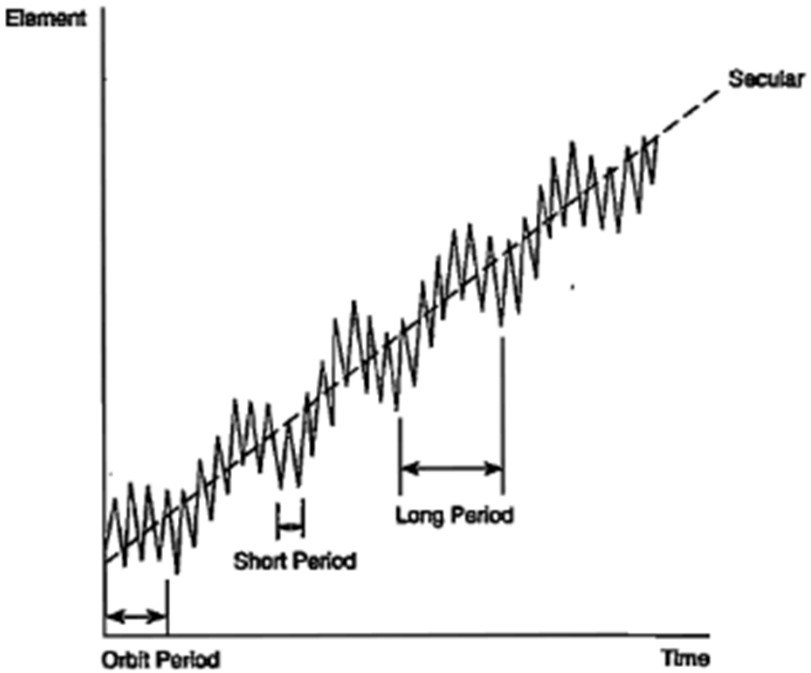
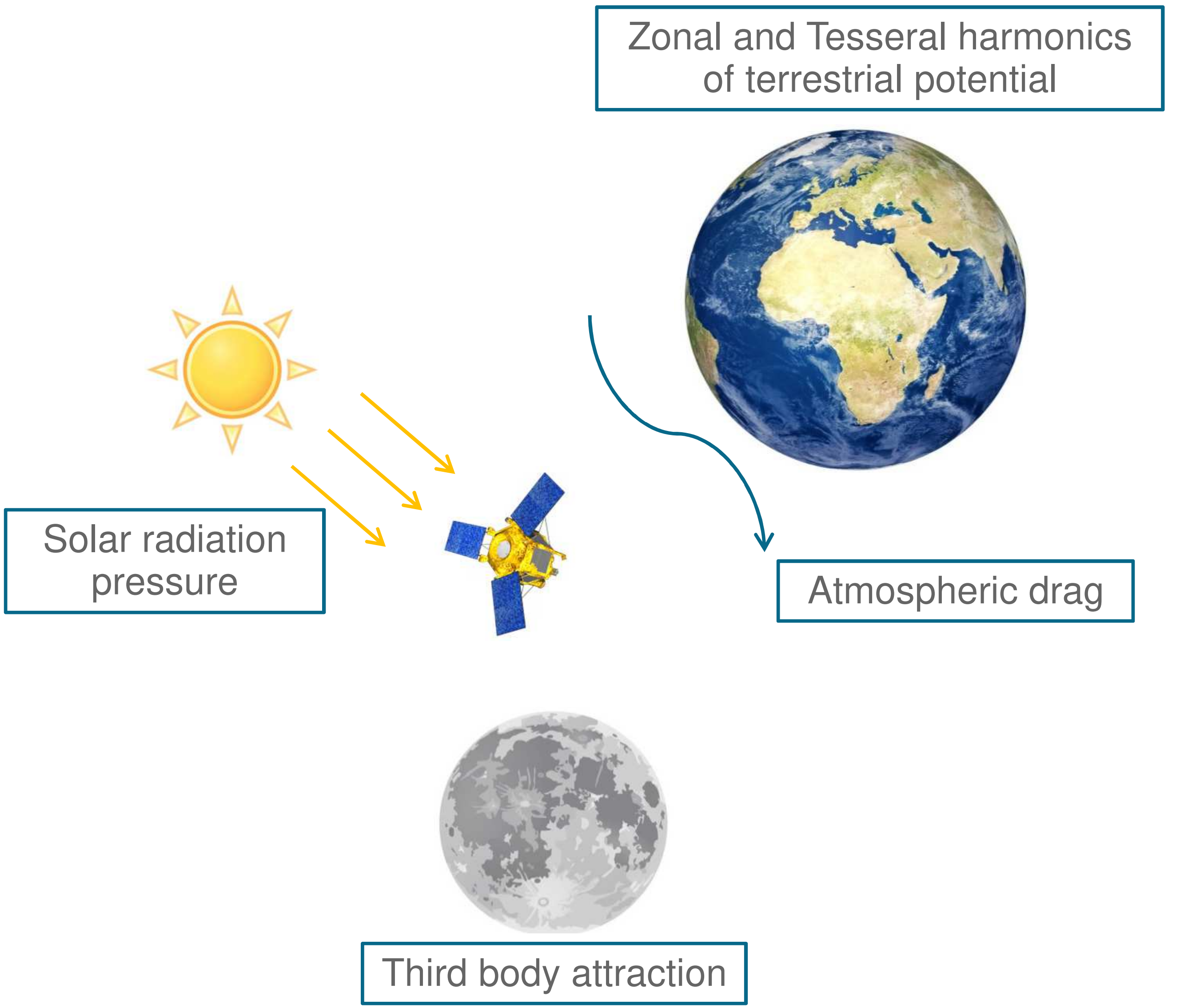
ORBITAL PERTURBATIONS





$$c_i(t) = \overline{c_i(t)} + \sum_{j=1}^{N} k_{ij}\eta_{ij}(t), \qquad i = 1, 2, 3, 4, 5, 6$$

Mean Elements

Short-periodic terms

DEVELOPMENT STEPS

Force models configuation

Computation of the mean elements derivatives and the short-periodic terms derivatives by automatic differentiation

Orbit Determination

Perform the DSST-OD with the Orekit's Batch Least Squares algorithm and the Kalman Filter.

Computation of the State Transition Matrices thanks to the variational equations

State Transition Matrices

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MEAN ELEMENTS DERIVATIVES

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MEAN ELEMENTS DERIVATIVES

➔ Each DSST-specific force model on Orekit has a method allowing the computation of the mean elements rates.

$$\mathbf{Y} = [\mathbf{a}\ \mathbf{h}\ \mathbf{k}\ \mathbf{p}\ \mathbf{q}\ \boldsymbol{\lambda}] \longrightarrow \dot{\mathbf{Y}} = \begin{bmatrix} \dot{\mathbf{a}} \\ \dot{\mathbf{h}} \\ \dot{\mathbf{k}} \\ \dot{\mathbf{p}} \\ \dot{\mathbf{q}} \\ \dot{\boldsymbol{\lambda}} \end{bmatrix}$$

➔ Method implemented for the states based on the real numbers ✔

➔ Need to be implemented to provide the Jacobians of the mean elements rates by automatic differentiation.

➔ GOAL

$$\begin{bmatrix} Y_i & \frac{\partial Y_i}{\partial Y_1} & \frac{\partial Y_i}{\partial Y_2} & \cdots & \frac{\partial Y_i}{\partial Y_6} & \frac{\partial Y_i}{\partial P_1} & \cdots & \frac{\partial Y_i}{\partial P_N} \end{bmatrix}$$

- **Y_i:** Orbital element
- **P_k:** Force model parameter
- **N:** The number of force model parameters taken into account for the Orbit Determination

➔ GAIN

- Safer implementation
- Simpler validation

JACOBIANS

$$\dot{\mathbf{Y}} = \begin{bmatrix} \dot{\mathbf{a}} \\ \dot{\mathbf{h}} \\ \dot{\mathbf{k}} \\ \dot{\mathbf{p}} \\ \dot{\mathbf{q}} \\ \dot{\boldsymbol{\lambda}} \end{bmatrix} \xrightarrow{\text{Automatic Differentiation}} \dot{\mathbf{Y}}' = \begin{bmatrix} \dot{\mathbf{Y}} & \dfrac{\partial \dot{\mathbf{Y}}}{\partial \mathbf{Y}} & \dfrac{\partial \dot{\mathbf{Y}}}{\partial \mathbf{P}} \end{bmatrix}$$

STATE TRANSITION MATRICES

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$$\dot{\mathbf{Y}}' = \begin{bmatrix} \dot{\mathbf{Y}} & \frac{\partial \dot{\mathbf{Y}}}{\partial \mathbf{Y}} & \frac{\partial \dot{\mathbf{Y}}}{\partial \mathbf{P}} \end{bmatrix}$$

Variational Equations

$$\frac{\mathbf{d}\left(\frac{\partial \mathbf{Y}}{\partial \mathbf{Y_0}}\right)}{\mathbf{dt}} = \frac{\partial \dot{\mathbf{Y}}}{\partial \mathbf{Y}} \times \frac{\partial \mathbf{Y}}{\partial \mathbf{Y_0}}$$

$$\frac{\mathbf{d}\left(\frac{\partial \mathbf{Y}}{\partial \mathbf{P}}\right)}{\mathbf{dt}} = \frac{\partial \dot{\mathbf{Y}}}{\partial \mathbf{Y}} \times \frac{\partial \mathbf{Y}}{\partial \mathbf{P}} + \frac{\partial \dot{\mathbf{Y}}}{\partial \mathbf{P}}$$

➔ Computation of $\frac{\partial Y}{\partial Y_0}$ and $\frac{\partial Y}{\partial P}$ matrices by finite differences and comparison to those previously obtained.

Problem : Different matrices !

➔ Newtonian Attraction derivatives were not taken into account in the computation of the state transition matrices.

➔ Some dependencies to the central attraction coefficient were implicit and therefore not differentiated.

Problem solved ✔

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SHORT-PERIODIC TERMS DERIVATIVES

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SHORT-PERIODIC TERMS DERIVATIVES

Automatic Differentiation

Compute the Jacobians of the short-periodic terms into the DSST-specific force models

Addition of the contribution

Add the contribution of the short-periodic derivatives after the numerical integration of the mean elements rates

Validation

Compute the state transition matrices (with the short-periodic derivatives) by finite differences

The user has the choice to use only the mean elements derivatives or adding the short-periodic terms derivatives

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ORBIT DETERMINATION

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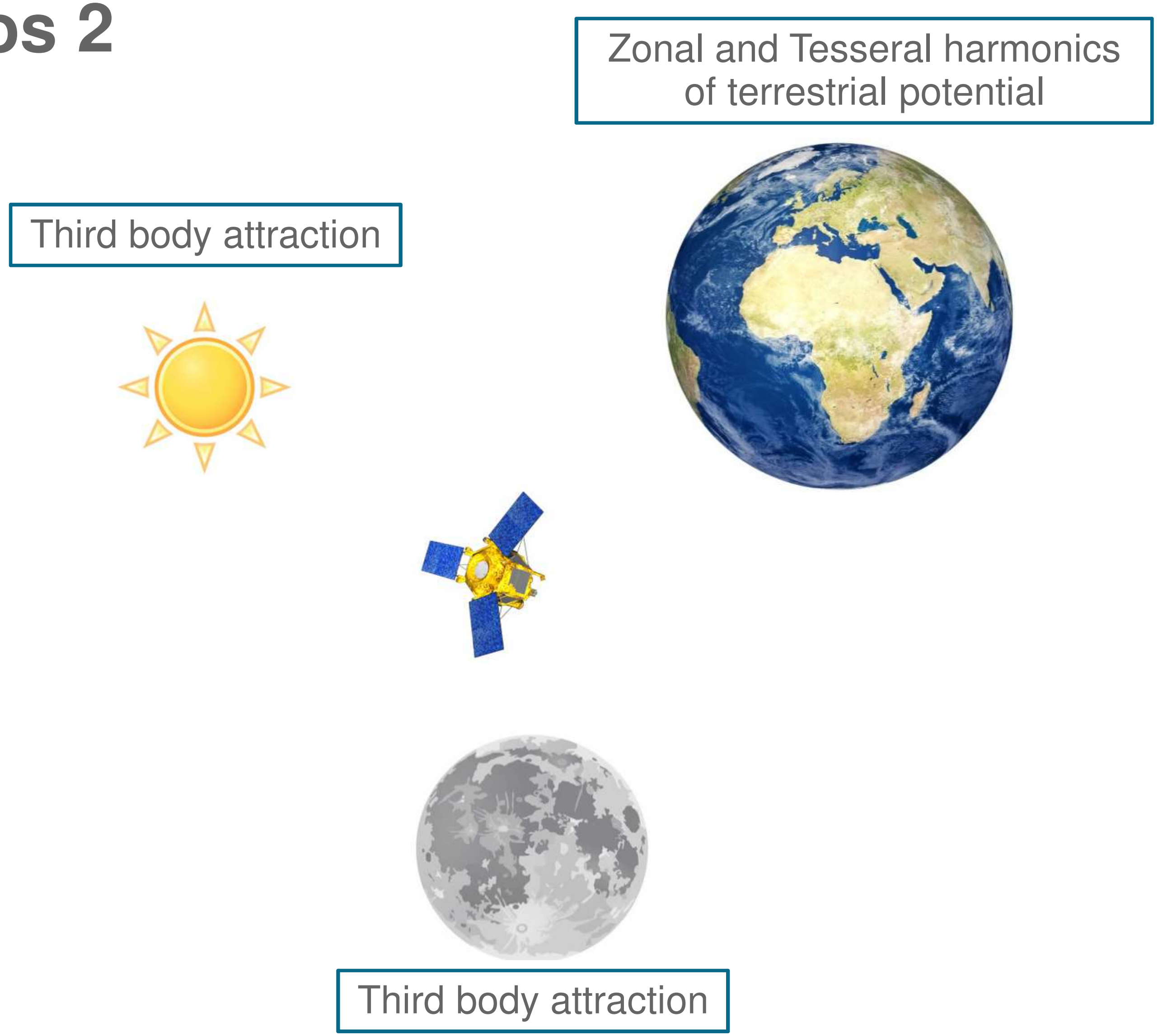
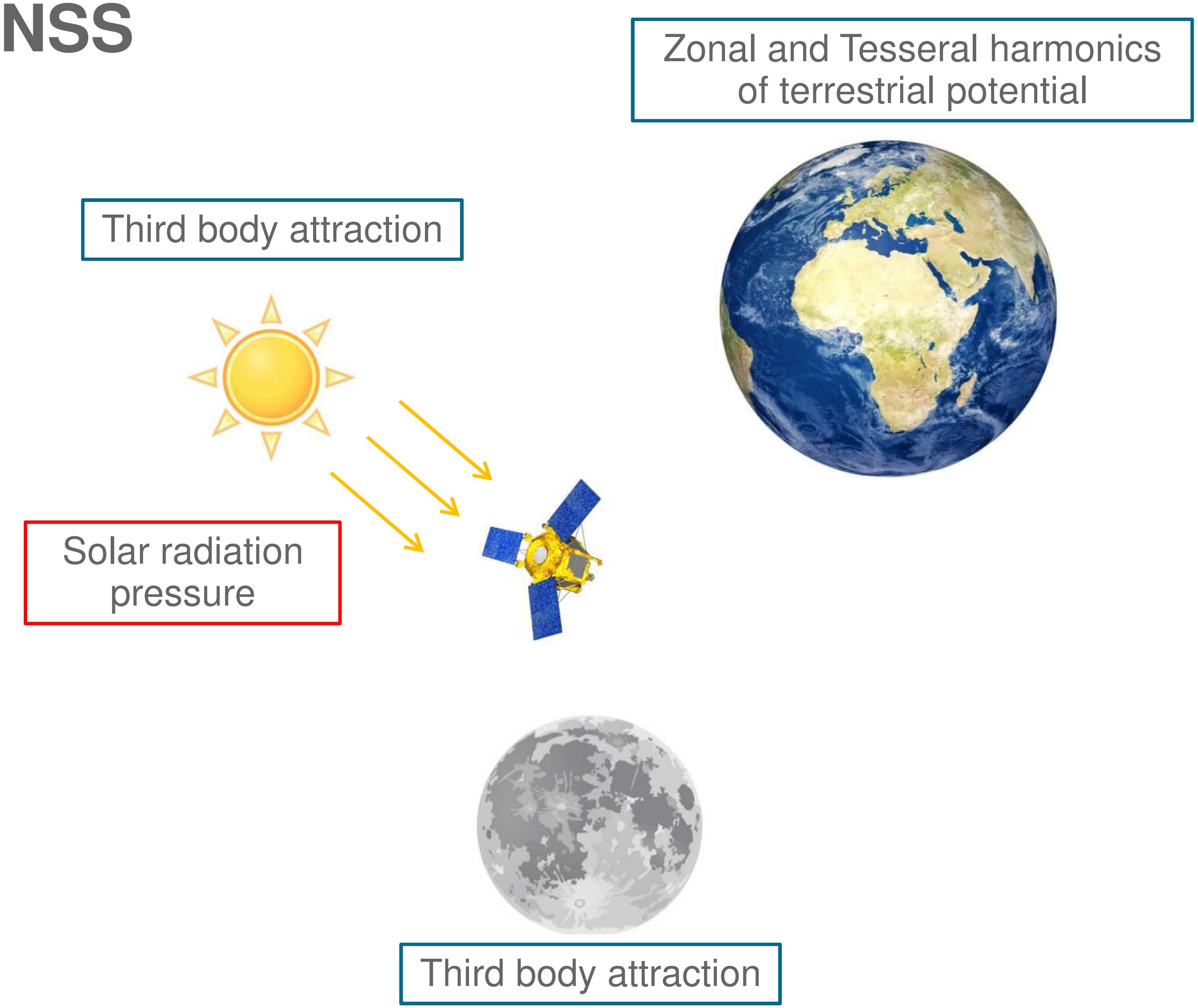
Lageos 2


Zonal and Tesseral harmonics of terrestrial potential
Third body attraction
Third body attraction

GNSS


Zonal and Tesseral harmonics of terrestrial potential
Third body attraction
Solar radiation pressure
Third body attraction

TEST CASES: INTEGRATION STEP

DSST

Minimum step (s)	6000
Maximum step (s)	86400
Tolerance (m)	10

VS

Numerical

Minimum step (s)	0,001
Maximum step (s)	300
Tolerance (m)	10

➔ The DSST has **significant advantage** compared to the numerical propagator for the integration step. This because the elements computed numerically by the DSST are the mean elements.

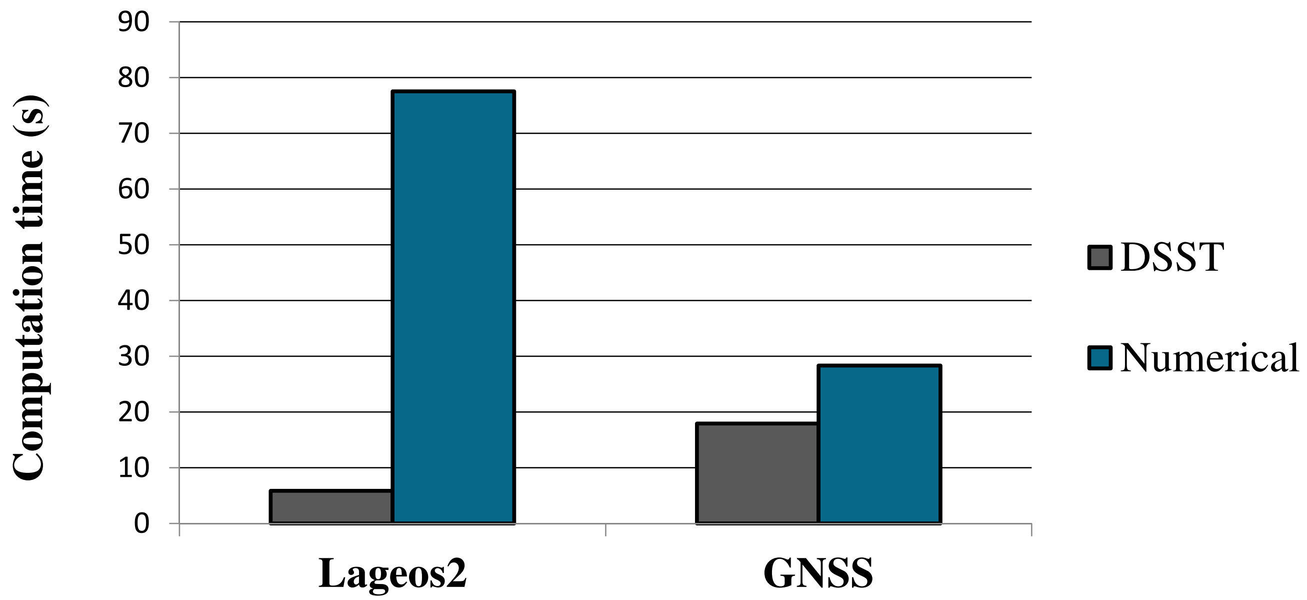
TEST CASES: SHORT-PERIODIC TERMS DERIVATIVES

Case	Zonal	Tesseral	Third Body
1	×	×	×
2	✓	×	×
3	✓	✓	×
4	✓	✓	✓

➔ Gradual addition of the short-periodic terms derivatives to highlight the main contributions.

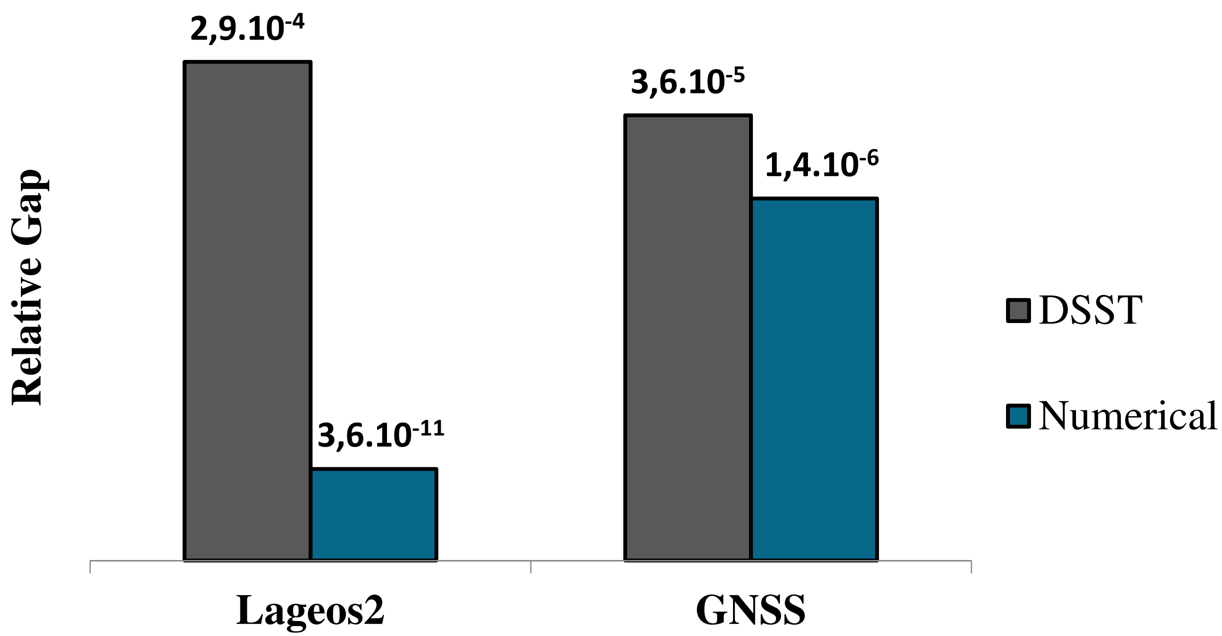
➔ Performed tests for Lageos2 Orbit Determination.

BATCH LEAST SQUARES / MEAN ELEMENTS

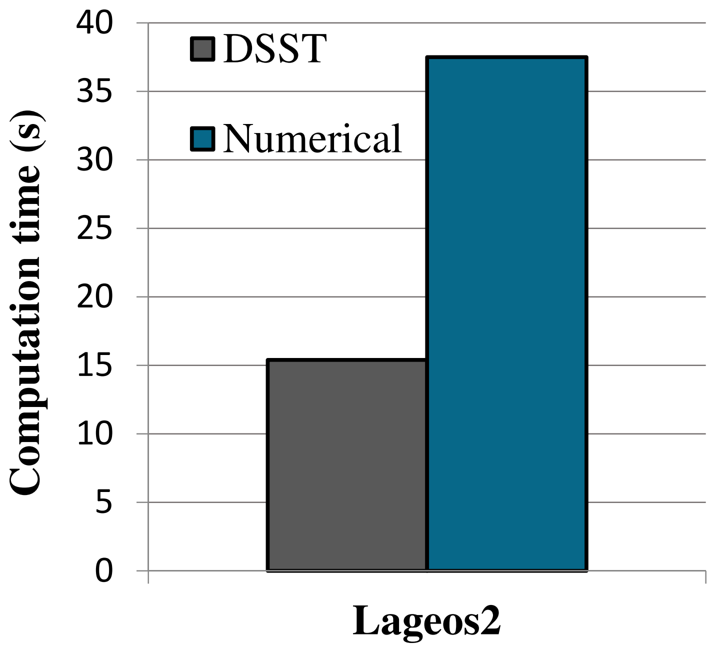


RAPIDITY

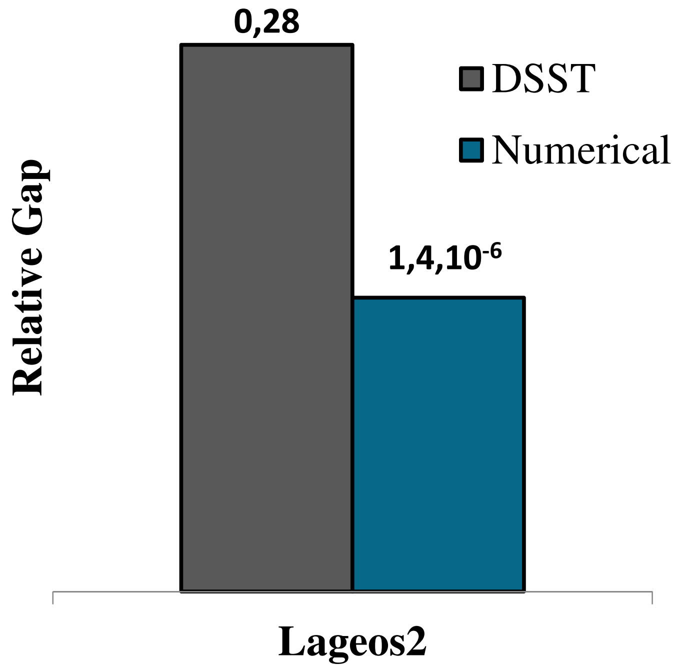
ACCURACY



KALMAN FILTER / MEAN ELEMENTS

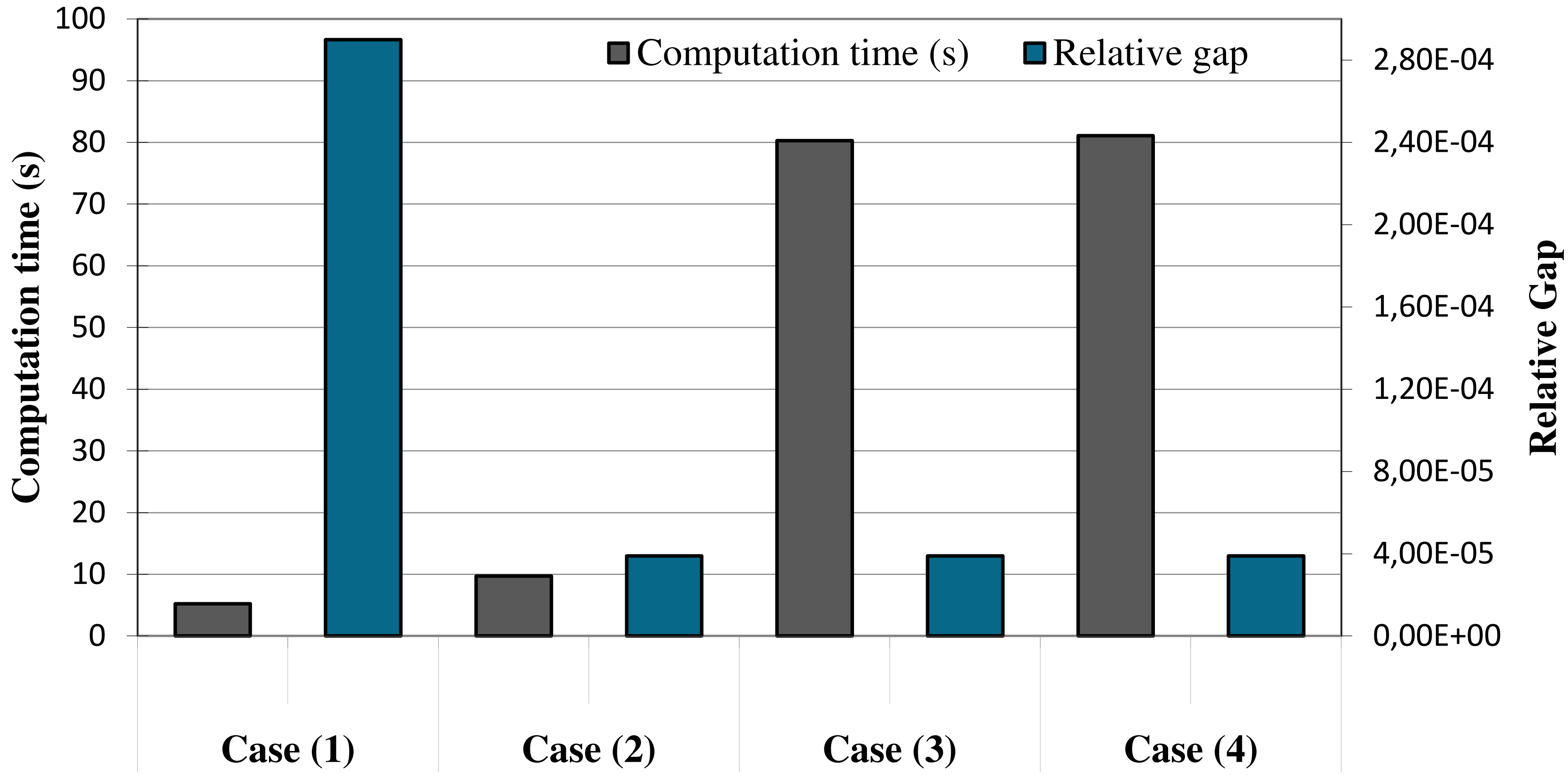


RAPIDITY

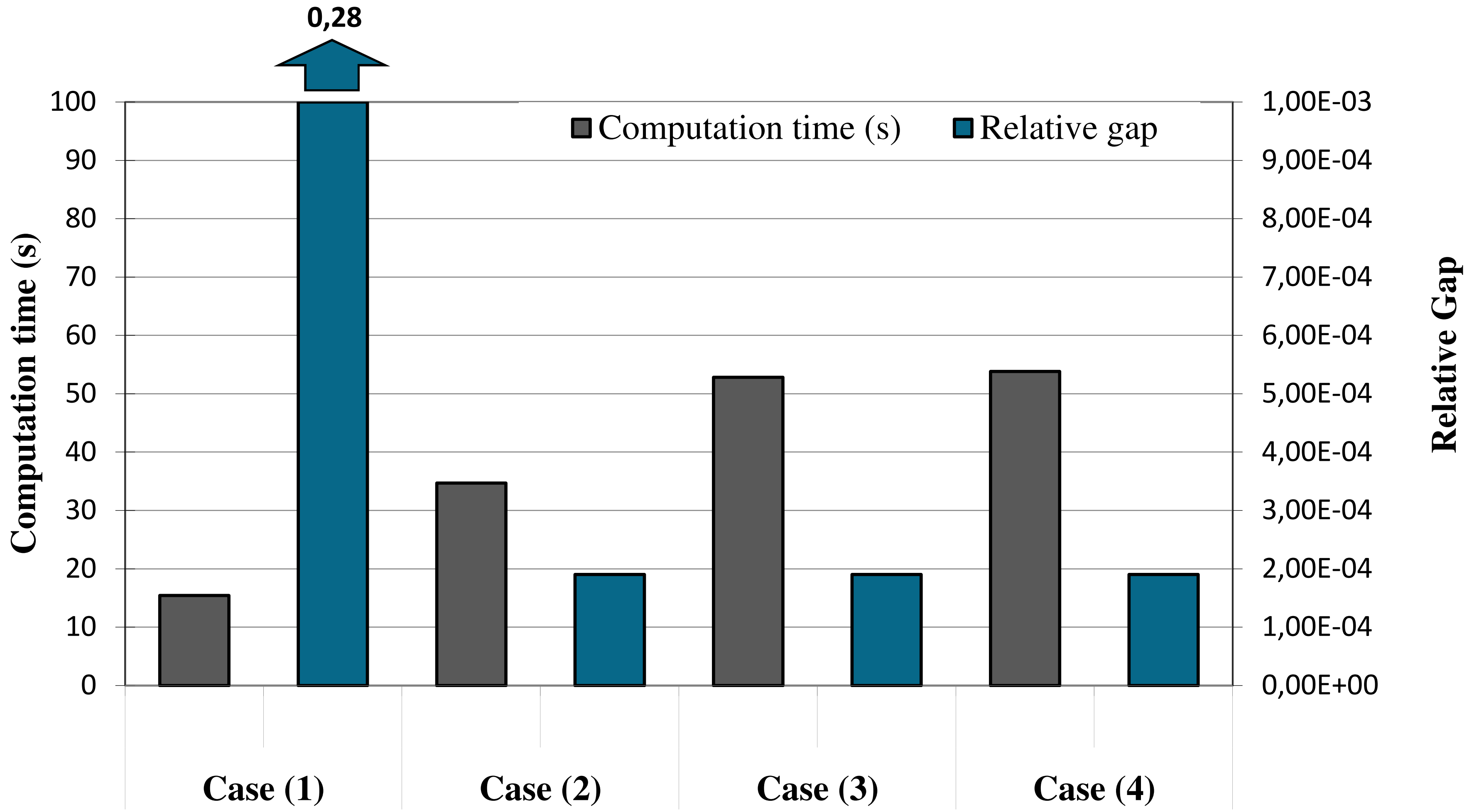


ACCURACY

BATCH LEAST SQUARES / LAGEOS2 SP DERIVATIVES



KALMAN FILTER / LAGEOS2 SP DERIVATIVES



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CONCLUSION

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OUTLOOKS

Need acces to « optimal » values for force models initialization.

Need to optimize critical computation steps.

DSST

Improve the Kalman Filter Orbit Determination with the DSST.

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Thank you for your attention

Publication : Open-source Orbit Determination using semi-analytical theory (Cazabonne et al, 2018)